Walsh **COMPLEX ANALYSIS** **Spring 2013**

MWF 9:00-9:50 AM, King 227

Instructor: Jim Walsh, King 220C
775-8387 (office); 775-8380 (messages)
(syllabus, homework assignments, handouts on Blackboard)

Office Hours:

Monday	3:00-4:00 PM
Tuesday	2:30-4:00 PM
Friday	2:00-3:00 PM

(*also by appointment*)

Text: J. Brown and R. Churchill, *Complex Variables and Applications*, 8th edition, McGraw-Hill, 2009. This required text is available at the College Bookstore.

On Reserve: (1) John B. Conway, *Functions of One Complex Variable*, 2nd edition, Springer-Verlag, 1978. (Also available in King 203.)
(2) Lars Ahlfors, *Complex Analysis*, 2nd edition (McGraw-Hill), 1966. (The third edition of this text is available in King 203.)

Homework: Homework problems will be collected on a regular basis during the course of the semester. You may work with other members of the class on all homework problems, but not to the extent you simply copy another's work (this would not help in the long run). You can also ask me about any and all problems in this course. If you do every assigned problem, collected or otherwise, you will likely do very well and you will learn a lot. Other than under *extraordinary* circumstances, *no late assignments will be accepted.*

Exams: There will be two midterm exams during the semester and a cumulative final exam. The due dates for the midterms are 13 March and 24 April. The final exam will take place 9:00-11:00 AM on 16 May.

Grading: Each of the two midterm exams counts 25% towards the final grade. The final exam and the homework contribute 30% and 20%, respectively, towards the final grade.

Honor System: You are urged to review the Honor Code and Honor System, available, for example, on the Blackboard site for this course. You will be expected to adhere to the Honor Code and Honor System with respect to all of your work in this class. One example: You may not use solutions to homework or exam questions found on the internet, or homework or exam solutions I have distributed for this course in the past. Another example: You may not copy any portion of the work of another student and submit it as your own.

Topics: The goal(!) is to cover much of Chapters 1-9 in Brown and Churchill, and parts of Chapters 6-7 in Conway. Time permitting, we also will learn a bit about Julia sets and the Mandelbrot set.

(*Very!*) Brief Historical Comments

- Babylonians (1700 BCE)

  Solve $x^2 + px + q = 0 \ \cdots \ x = \sqrt{\left(\frac{p}{2}\right)^2 - q} - \frac{p}{2}$

  Ex. Solve $x^2 + 4x + 13 = 0$. This yields $x = \sqrt{(2)^2 - 13} - 2 = -2 + 3\sqrt{-1}$. *How does one interpret this expression in x?*

- Cardano (1501–1576) Authored *Ars Magna* (1545)

  Dismissing mental tortures, and multiplying $5+\sqrt{-15}$ *by* $5-\sqrt{-15}$, *we obtain* $25-(-15)$. *Therefore the product is 40. ... and thus far does arithmetical subtlety go, of which this, the extreme, is, as I have said, so subtle that it is useless.*

  (Cardano also wrote: *This I recognise as unique and outstanding amongst my faults—the habit, which I persist in, of preferring to say above all things what I know to be displeasing to the ears of my hearers. I am aware of this, yet I keep it up wilfully, in no way ignorant of how many enemies it makes for me.*)

- Leibniz (1646–1716)

  Showed $\sqrt{6} = \sqrt{1 + \sqrt{-3}} + \sqrt{1 - \sqrt{-3}}$.

  Leibniz wrote: *Complex numbers are like the Holy Ghost (or amphibians)—halfway between existence and nonexistence.*

- Euler (1707–1783)

  Introduced notation $a + i\,b$, $a, b \in \mathbb{R}$, $i = \sqrt{-1}$, in 1777

  Made no use of geometry

- Wessel (1745-1818)

  Norwegian-born surveyor who in a 1799 paper introduced the idea of a complex number as a point or vector in the plane.

- Cauchy (1789–1857) 'Created' complex function theory in years 1814–1831

For me, as a Baconian, the main thing missing in the Bourbaki program is the element of surprise. The Bourbaki program tried to make mathematics logical. When I look at the history of mathematics, I see a succession of illogical jumps, improbable coincidences, jokes of nature. One of the most profound jokes of nature is the square root of minus one that the physicist Erwin Schrödinger put into his wave equation when he invented wave mechanics in 1926. Schrödinger was a bird who started from the idea of unifying mechanics with optics. A hundred years earlier, Hamilton had unified classical mechanics with ray optics, using the same mathematics to describe optical rays and classical particle trajectories. Schrödinger's idea was to extend this unification to wave optics and wave mechanics. Wave optics already existed, but wave mechanics did not. Schrödinger had to invent wave mechanics to complete the unification.

Starting from wave optics as a model, he wrote down a differential equation for a mechanical particle, but the equation made no sense. The equation looked like the equation of conduction of heat in a continuous medium. Heat conduction has no visible relevance to particle mechanics. Schrödinger's idea seemed to be going nowhere. But then came the surprise. Schrödinger put the square root of minus one into the equation, and suddenly it made sense. Suddenly it became a wave equation instead of a heat conduction equation. And Schrödinger found to his delight that the equation has solutions corresponding to the quantized orbits in the Bohr model of the atom.

It turns out that the Schrödinger equation describes correctly everything we know about the behavior of atoms. It is the basis of all of chemistry and most of physics. And that square root of minus one means that nature works with complex numbers and not with real numbers. This discovery came as a complete surprise, to Schrödinger as well as to everybody else. According to Schrödinger, his fourteen-year-old girl friend Itha Junger said to him at the time, "Hey, you never even thought when you began that so much sensible stuff would come out of it." All through the nineteenth century, mathematicians from Abel to Riemann and Weierstrass had been creating a magnificent theory of functions of complex variables. They had discovered that the theory of functions became far deeper and more powerful when it was extended from real to complex numbers. But they always thought of complex numbers as an artificial construction, invented by human mathematicians as a useful and elegant abstraction from real life. It never entered their heads that this artificial number system that they had invented was in fact the ground on which atoms move. They never imagined that nature had got there first.

–Freeman Dyson, "Birds and Frogs," *Notices of the AMS* **56 (2) (2009)**