Optimization

TuTh
3:00-4:20
SCTR A255

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Office hours

M–Th 4:30-5:30 and by appointment.

Course overview

Optimization is the branch of mathematics that deals with optimal performance—finding the best way to complete a task. Most of us saw it first in Calculus I, where we learned how to solve problems like

> **The single-variable, unconstrained problem:** Given a function $f : I\!R \to I\!R$, find $x^* \in I\!R$ such that $f(x^*) \leq f(x)$ for all $x \in I\!R$.

> **The single-variable, constrained problem:** Given a function $f : I\!R \to I\!R$ and an interval $S = [a, b]$, find $x^* \in S$ such that $f(x^*) \leq f(x)$ for all $x \in S$.

and to tackle tasks like

> **Task 1:** Design a cylindrical can to hold one liter of oil.

> **Task 2:** Determine the dimensions of a rectangular garden, 450 ft^2 in area, to be fenced off from rabbits. One side will be the barn wall, impermeable to rabbits.

Question: For each task, what does "optimal performance" mean?

... ...

Most of us had our next encounter with optimization in Calculus III:

> **The multivariable, unconstrained problem:** Given a function $f : I\!R^n \to I\!R$, find $\boldsymbol{x}^* \in I\!R^n$ such that $f(\boldsymbol{x}^*) \leq f(\boldsymbol{x})$ for all $\boldsymbol{x} \in I\!R^n$.

> **The multivariable, equality-constrained problem:** Given functions $f : I\!R^n \to I\!R$ and $g_i : I\!R^n \to I\!R$, $1 \leq i \leq m$, together with constants c_i, $1 \leq i \leq m$, define the set $S = \{\boldsymbol{x} \in R^n : g_i(\boldsymbol{x}) = c_i \text{ for all } 1 \leq i \leq m\}$ and then find $\boldsymbol{x}^* \in S$ such that $f(\boldsymbol{x}^*) \leq f(\boldsymbol{x})$ for all $\boldsymbol{x} \in S$.

Note: Here, and for the rest of this document, $\boldsymbol{x}^* = (x_1^*, \ldots, x_n^*)^T$ and $\boldsymbol{x} = (x_1, \ldots, x_n)^T$.

... ...

For the present course, we will stay in the multivariable world and study optimization problems of the form

$$\begin{array}{ll} \text{minimize} & f(\boldsymbol{x}) \\ \text{subject to} & \boldsymbol{x} \in S, \end{array}$$

where

$$S = \{\boldsymbol{x} \in I\!R^n \;:\; \begin{array}{ll} g_i(\boldsymbol{x}) = c_i & \text{for all } 1 \leq i \leq m \\ h_i(\boldsymbol{x}) \leq d_i & \text{for all } 1 \leq i \leq p\} \end{array}$$

(the continuous case) or

$$S = \{\boldsymbol{x} \in Z^n \;:\; \begin{array}{ll} g_i(\boldsymbol{x}) = c_i & \text{for all } 1 \leq i \leq m \\ h_i(\boldsymbol{x}) \leq d_i & \text{for all } 1 \leq i \leq p\} \end{array}$$

(the discrete case). We will see that if the functions f, g_i, and h_i are "well behaved," the problems can be "easy." If not, they seem to be extremely "hard." We will also see that the continous case is usually easier (and often *much* easier) than the discrete case.

... ...

Throughout the semester, we will focus on concepts, mechanics, and modeling. One of my goals is to help you appreciate the beauty of the theory underlying this subject. Another is to help you become adept at the mechanics of executing the algorithms. The third—and perhaps most important—is to help you become proficient in the art of mathematical modeling, which entails taking a "real-world" problem, identifying its most important features, and representing them mathematically in a (hopefully) tractable form.

Required book

William J. Cook's *In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation.* This book is not a text. Instead of requiring a text, I will distribute numerous handouts.

Homework

Most homework assignments will be a mix of conceptual exercises, mechanical exercises, and exercises that require the use of a computer. I encourage you to discuss the assignments with your classmates, but I insist that you write and submit your own solutions. Absolutely no late homework will be accepted without a valid excuse (an illness, an emergency, etc.). Homework must be turned in (to my office) by 5:00 pm on Fridays.

Homework grading

I'm hoping that two or three students will volunteer each week to help me grade the week's homework. We will meet at lunch time on Saturday or Sunday and grade for no more than two hours. The benefits of volunteering are as follows: (1) you will be fed, (2) you will get your lowest two homework scores dropped (instead of just one), (3) you will be able to see how your classmates tackled the problems and how they wrote them up. (For several reasons, this third benefit tends to raise the quality of everyone's homework papers.)

Midterm Exams

There will be two open-text, open-notes take-home exams. Tentative distribution dates are October 3 and November 21.

Final project

The final project will give you the opportunity to do some modeling, to do some programming, or to write a research paper on a topic that interests you. Each **team** of 2–4 students must give a 10–15 minute presentation of their project and submit a 5–10 page written report. Presentations will be held during the last week of class. Hard copies of the written reports will be collected at 9 pm on Friday December 20. I will point out possible final project topics throughout the semester.

Final exam

If you do not want to complete a final project, you may take a final exam (which must be handed in by 9pm on Friday December 20).

Grading

The homework assignments will be worth a total of 25% of the final grade. Each exam will be worth 25%. The final project will be worth 25%.

Cell phones

When you come to class, please silence your cell phones. Setting them on vibrate is fine. If your phone rings in class, you will be expected to bring food for the entire class the next time we meet.

Help

Please feel free to ask me questions about the course (or anything else). If you find my office hours inconvenient, you are welcome to schedule an appointment (or just drop by).

My Research

Optimization is my favorite branch of mathematics and my favorite course to teach. The subject matter is closely tied to my research, so on multiple occasions during the semester I will talk about my research and the research I've done with students who've taken this class.

Publications

R.A. Bosch and N.K. Stout (**OC 1994**), "Refinements of Paltiel and Kaplan's decision-theoretic model of AIDS clinical trials," *Socio-Economic Planning Sciences* 31 (1997) 87-102.

R.A. Bosch and J.A. Smith (**OC 1996**), "Separating hyperplanes and the authorship of the disputed Federalist papers," *The American Mathematical Monthly*, 105 (1998) 601-608.

M. Hart (**OC 2000**), R.A. Bosch, and E. Tsai (**OC 1999**), "Finding optimal piano fingerings," *The UMAP Journal*, 21(2) (2000) 167-177.

M. Cardiff (**OC 2001**), G. Hughes (**OC 2001**), and R. Bosch, "Maximizing fun at a theme park," *The UMAP Journal* 21(4) (2000) 483-498.

R. Bosch and A. Herman (**OC 2004**), "Continuous line drawings via the traveling salesman problem," *Operations Research Letters* 32 (2004) 302-303.

R. Bosch and A. Herman (**OC 2004**), "Pointillism via linear programming," *The UMAP Journal* 26(4) (2005) 393-400.

R Bosch and A. Pike (**OC 2008**), "Map-colored mosaics," *Conference Proceedings of Bridges: Mathematical Connections in Art, Music, and Science* (2009) 139-146.

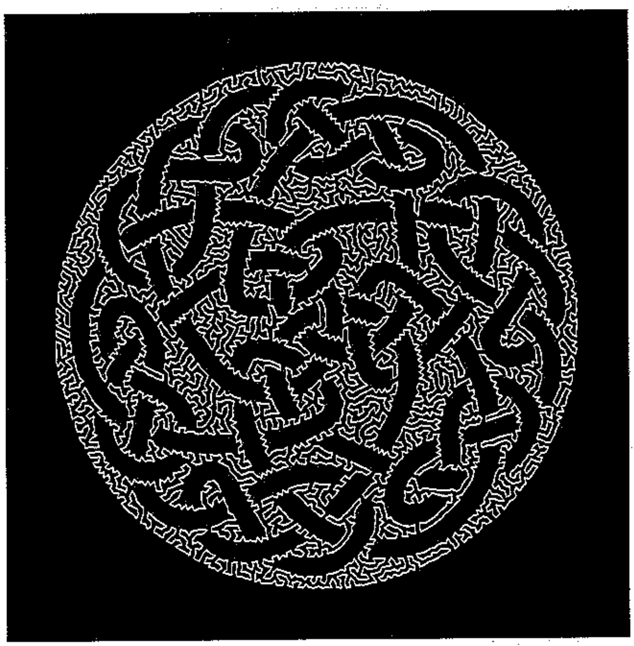

Figure 1: *Knot?* 2006 (117 ft curve on 34 by 34 in canvas)