Mathematics 302 **DYNAMICAL SYSTEMS** Spring 2014

MWF 11:00-11:50 AM, King 237

Yea verily, I say unto you:
A person must have Chaos yet within him
To birth a dancing star.
I say unto you:
You have yet Chaos in you.

– Friedrich Nietzsche

Instructor: J. A. Walsh, King 220D
775-8387 (office); 775-8380 (messages)
(syllabus, homework assignments, handouts on Blackboard)

Office Hours:

Monday	3:00-4:30 PM	
Tuesday	2:30-4:00 PM	
Friday	2:00-3:00 PM	(*also by appointment*)

Required Text: R.L. Devaney, *An Introduction to Chaotic Dynamical Systems*, 2nd edition, Westview Press (2003). This required text is available at the College Bookstore.

Texts on Reserve in Mudd:

1. K. Alligood, T. Sauer & J. Yorke, *Chaos: An Introduction to Dynamical Systems*[1,2], Springer (2000).

2. M. Barnsley, *Fractals Everywhere*, 2nd edition, Academic Press (1993).

3. B. Davies, *Exploring Chaos: Theory and Experiment*[1], Westview Press (2004).

4. R.L. Devaney, *A First Course in Chaotic Dynamical Systems: Theory and Experiment*[1], Addison-Wesley (1992).

5. S. Elaydi, *Discrete Chaos*[1], Chapman & Hall/CRC Press (2000).

6. B. Hasselblatt & A. Katok, *A First Course in Dynamics: With a Panorama of Recent Developments*[1], Cambridge University Press (2003).

7. M. Hirsch et al, *Differential Equations, Dynamical Systems, and an Introduction to Chaos*[2], Elsevier/Academic Press (2004).

8. R. Holmgren, *A First Course in Discrete Dynamical Systems*[1], 2nd edition, Springer-Verlag (1996).

9. L. Perko, *Differential Equations and Dynamical Systems*[2], Springer-Verlag (1991).

10. R.C. Robinson, *An Introduction to Dynamical Systems: Continuous and Discrete*[1,2], Prentice Hall (2004).

11. S. Strogatz, *Nonlinear Dynamics and Chaos: With Applications to Physics, Biology, Chemistry, and Engineering*[2], Addison-Wesley (1994).

12. A. Taylor & W. Mann, *Advanced Calculus*, Wiley (1983).

13. E. Izhikevich, *Dynamical Systems in Neuroscience: The Geometry of Excitability and Bursting*, MIT Press (2010).

[1]Have considered using (or have used!) as a text for this course.
[2]For those having a background in differential equations.

Popular Books on Chaos:

1. R. Abraham & Y. Ueda (eds), *The chaos avant-garde: memories of the early days of chaos*, World Scientific (2000).
2. F. Diacu & P. Holmes, *Celestial encounters*, Princeton University Press (1996).
3. L. Glass & M. Macke, *From Clocks to Chaos: The Rhythms of Life*, Princeton University Press (1988).
4. J. Gleick, *Chaos: Making a New Science*, Penguin (1988).
5. I. Stewart, *Does God Play Dice? The Mathematics of Chaos*, Basil Blackwell (1989).

Software:

You will spend some time this semester investigating the behavior of dynamical systems via numerical simulations. There are many applications on the web you might use, including those found at:

(i) http://math.bu.edu/DYSYS/ (a variety of tools for iterating maps)

(ii) http://www.clickrepair.net/chaos/ (a user's manual for the application *Chaos for Java* can be found at this site as well as on page 199 in the book *Exploring Chaos* by Davies, on reserve in Mudd)

Spreadsheets and the applications *Mathematica* and *Matlab* can be useful if you have some facility with any of these. *If you find a good discrete dynamics web application please let me know!*

Goals/ Outline:

The goal of this course is to present a first introduction to the field of dynamical systems. You will investigate some of the recent work and discoveries in dynamics in our class meetings, via computer experiments and, perhaps, through your research projects (see below). No computer experience is required for this course.

Because dynamical systems is such a burgeoning field, of interest to physicists, biologists, chemists and economists as well as mathematicians, there is much more interesting, accessible material than we can hope to cover in one semester. This course is an introduction to the discrete side of the field. Below are selected topics that, time permitting, we will cover in some depth. (*Note:* It is likely we will not cover everything listed below!).

I. One-Dimensional Dynamics

We will begin by covering selected topics in Chapter 1 in Devaney. Many of the ideas and techniques of nonlinear dynamics can be introduced in the one-dimensional setting. Topics not covered in Chapter 1 may serve as a catalyst for your research project.

II. Two-Dimensional Dynamics (following Chapter 2 in Devaney)

- Dynamics of linear maps
- Smale's horseshoe map
- Stable and Unstable Manifold Theorem
- The Hénon map

III. Complex Dynamics (following parts of Chapter 3 in Devaney and Chapters 15-17 in Devaney's *A First Course in Chaotic Dynamical Systems*, Addison-Wesley, 1992.)

- A brief survey of complex analysis
- Julia sets
- The Mandelbrot Set

IV. Fractals (following the approach in *Fractals Everywhere* by Michael Barnsley, Academic Press, 1988.)

- Iterated Function Systems and the Collage Theorem
- Fractal dimension

Grading: There will be two midterm exams, with dates of 5 March and 9 April. Each midterm exam counts as one-fourth of your final grade.

Homework will be assigned on a regular basis. Selected problems will be collected and graded. You may work in groups (and are encouraged to do so) on these problems, but not to the point of simply copying another's work. In addition, you will be required to hand in a few lab reports. The homework and lab reports comprise one-fourth of the final grade.

Except under extraordinary circumstances, late assignments will not be accepted.

Honor System: You are urged to review the Honor Code and Honor System, available, for example, on the Blackboard site for this course. You will be expected to adhere to the Honor Code and Honor System with respect to all of your work in this class. One example: You should not use solutions to homework or exam questions found on the internet or from homework or exam solutions I have distributed for this course in the past. Another example: You may not copy any portion of the work of another student and submit it as your own.

Research Project: One-fourth of your final grade will be based on a research project, including an expository paper and an in-class presentation. **The paper is due by 9:00 PM on 15 May**. You may work with a partner on the project if you so desire. Expectations for this project are high—you are expected to show initiative in learning dynamical systems material, and you are expected to convey what you have learned to us.

You are free to choose a topic from a personal area of interest, as long as there is a firm connection to this course. You may not duplicate research you are doing or have done in any other setting. You might consult the relevant literature, or perhaps choose a topic we have not covered in class from our text or one of the books placed on reserve. I will of course be available for advice or aid in working through your chosen topic.

I suggest that you begin your research as soon as you feel you have sufficient background from this course. When ready to do so, you might make a trial run of your presentation to friends. You will be graded on the ambitiousness of the topic, the quality of your presentation and expository paper, and the amount of mathematics learned.

A one-page project prospectus is due by 11:00 AM on 7 April at the latest. Five points will be deducted from your project grade if the prospectus is not submitted by this deadline.

Henri Poincaré
1854–1912

So, Nat'ralists observe, a Flea
Hath smaller Fleas that on him prey,
And these have smaller Fleas to bite 'em,
And so proceed ad infinitum.

– Jonathan Swift

Big whorls have little whorls
Which feed on their velocity;
And little whorls have lesser whorls,
And so on to viscocity.

–Lewis Richardson, parodying Swift's doggerel.